Assignment 8.

1. Solve the equation

$$u_t + \frac{\sigma^2 x^2}{2} u_{xx} + r x u_x - r u = 0 \quad \text{for} \quad s \le t \le T$$

with

$$u(T,x) = (x-c)^+ = \begin{cases} x - c & \text{if } x \ge c \\ 0 & \text{otherwise} \end{cases}$$

explicitly in terms of the tails of a normal distribution.

2. Feynman-Kac formula. If V is a bounded function and $\beta(t)$ is Brownian motion, by expanding the exponential inside the expectation in

$$u(t,x) = E_x[f(x(t)) \exp[\int_0^t V(x(s))ds]$$

get an infinite series representation for u. Differentiate term by term to show

$$u_t(t,x) = \frac{1}{2} u_{xx}(t,x) + V(x)u(t,x)$$

[Hint: Write the n-th power of the integral as a multiple integral over as n times, and then as $n!$ times a time ordered integral.]