Assignment 6.

1. Assume a family $p_h(dy)$ of probability distributions on R^d satisfy

$$\lim_{h\to 0}\frac{1}{h}\int y_i p_h(dy) = b_i$$

$$\lim_{h\to 0}\frac{1}{h}\int y_i y_j p_h(dy) = a_{i,j}$$

and for some $\delta > 0$

$$\lim_{h\to 0}\frac{1}{h}\int \|y\|^{2+\delta} p_h(dy) = 0$$

Conclude that for any smooth function $f(y)$

$$\lim_{h\to 0}\frac{1}{h}\int [f(y)-f(0)]p_h(dy) = \frac{1}{2}\sum_{i,j} a_{i,j}\big(\frac{\partial^2 f}{\partial x_i \partial x_j}\big)(0) + \sum_j b_j\big(\frac{\partial f}{\partial x_j}\big)(0)$$

2. Show that the transition probability $p(s,x,t,dy)$ of the solution $x(t)$ to the SDE $dx = bdt + \sigma d\beta$ satisfies

$$\lim_{h\to 0}\frac{1}{h}\int [f(y)-f(x)]p(t,x,t+h,dy) = \frac{1}{2}\sum_{i,j} a_{i,j}(t,x)\big(\frac{\partial^2 f}{\partial x_i \partial x_j}\big)(x) + \sum_j b_j(t,x)\big(\frac{\partial f}{\partial x_j}\big)(x)$$