Assignment 3.

Exercise 1. In the case of Brownian motion, if τ is a stopping time that takes only a countable number of values show that the process $x(\tau + t) - x(\tau)$ is again a Brownian motion independent of $\mathcal{F}_\tau$.

Exercise 2. If τ is a stopping time, then show that $\tau_n = \frac{[n\tau]+1}{n}$ where $[x]$ is the largest integer not exceeding x, is again a stopping time. Using the fact that $\tau_n \geq \tau$ and $\tau_n \downarrow \tau$ as $n \to \infty$, extend the strong Markov property for Brownian motion to any stopping time τ with $P[\tau < \infty] = 1$.