Assignment 1.

1. There are two coins. Coin I and coin II. Coin I has probability $\frac{3}{4}$ for coming up head and $\frac{1}{4}$ for tail. Coin II on the other hand will come up head with probability $\frac{1}{4}$ and tail with probability $\frac{3}{4}$. We have a sequence of tosses. If at a given toss it comes up heads the coin is retained for the next toss. If it comes up tails the coin is switched. S_n is the number of heads in the first n tosses. Assume that Coin I is used for the first toss. Let $X_i = 1$ if the i-th toss is head and 0 if it is a tail. Then $S_n = X_1 + X_2 + \cdots + X_n$. Find functions $f_n(X_1, X_2, \ldots, X_n)$ such that $S_{n+1} - \sum_{i=1}^{n} f_i(X_1, X_2, \ldots, X_i)$ is a Martingale with respect to the σ-fields $\sigma(X_1, \ldots, X_n)$.

2. Do problem 6 from

http://www.math.nyu.edu/faculty/varadhan/stochastic.fall08/4.pdf

with the justification asked for in problem 7.