Problemset 2. Due April 11.

The Gamma process is defined as a process with independent and stationary increments whose distribution at time 1 is the exponential distribution with density

$$p(x) = \begin{cases} e^{-x}dx & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

1) What is its distribution at time $t > 0$?

2) What is the Levy-Khintchine representation for the process?

3) Show that the process is increasing and made up only of positive jumps.

4) What is the distribution of the biggest jump during $0 \leq t \leq 1$?

5) Given $X(1) \geq A$ what is $EX(1)$?

6) If A is large, show that the large value of $X(1)$ is due to at least one jump of order of magnitude A i.e if $\Omega_{\delta A}$ is the set of paths with no jumps of size larger than δA, then

$$\lim_{\delta \to 0} \limsup_{A \to \infty} P[\Omega_{\delta A} | X(1) = A] = 0$$

7) If Y is the largest jump can you calculate asymptotically the conditional distribution of $\frac{Y}{X(1)}$ given $X(1) = A$, asymptotically as $A \to \infty$?

8) At what point in time would the largest jump have occurred?