Problem set 4.

1. If $d(x,y)$ is a metric on X show that so is $D(x,y) = \frac{d(x,y)}{1+d(x,y)}$. Show that (X,d) is complete if and only if (X,D) is. Show that they have the same collection of open sets.

2. $X = \Pi_{i=1}^{\infty} R$. $\xi \in X$ is a sequence of real numbres

$$\xi = \{x_1, x_2, \ldots, x_n, \ldots\}$$

We say that sequence $\xi_n = \{x_j^n\}$ converges to $\xi = \{x_j\}$ if for every j, $\lim_{n\to\infty} x_j^n = x_j$. Can you construct a metric that corresponds to this convergence. Is it complete?

3. Is the space X with this metric separable? Can you describe a countable basis for the open sets?