

Assignment 8. Due Nov 11, 2003

Q 1. For $p \geq 1$, l_p is the space of sequences $a = \{a_n\}$ with $\sum |a_n|^p < \infty$. Check that $\|a\|_p = (\sum_n |a_n|^p)^{\frac{1}{p}}$ is a norm on l_p and l_p is complete under this norm.

Q 2. If $\lambda \ll \mu$ and $\mu \ll \lambda$ with $\phi = \frac{d\lambda}{d\mu}$ show that $\phi > 0$ a.e. λ (as well as μ) and

$$f \rightarrow \phi^{\frac{1}{p}} f$$

provides an isometry between $L_p(\lambda)$ and $L_p(\mu)$.