Assignment 5. Due Oct 21, 2003

Q 1. We say that F is absolutely continuous in $[0,1]$ if for any given $\epsilon > 0$, there exists a $\delta > 0$ such that

$$\sum_{j=1}^{N} |F(b_j) - F(a_j)| \le \epsilon$$

when ever the intervals $[a_j, b_j] \subset [0,1]$ are disjoint and

$$\sum_{j=1}^{N} |b_j - a_j| < \delta.$$

Show that if F is absolutely continuous then it is continuous and is of bounded variation. If $F(x) = F_1(x) - F_2(x)$ is its minimal decomposition as the difference of two nondecreasing functions then show that F_1 and F_2 are absolutely continuous.

Q 2. Deduce from **Q 1** that F is absolutely continuous on $[0,1]$ if and only if

$$F(x) = F(0) + \int_0^x f(y)dy$$

for some integrable f.