Assignment 11. Due Dec 2, 2003.

Q1. Consider the space ℓ_1 of sequences $\mathbf{a} = \{a_n : n \geq 1\}$ such that $\sum_{n=1}^{\infty} |a_n| = \|\mathbf{a}\| < \infty$. The distance between two sequences $\mathbf{a}$ and $\mathbf{b}$ is defined as

$$d(\mathbf{a}, \mathbf{b}) = \sum_{n=1}^{\infty} |a_n - b_n|$$

Show that a closed subset $C \subset \ell_1$ is compact if and only if

$$\sup_{\mathbf{a} \in C} \|a\| < \infty$$

and

$$\lim_{N \to \infty} \sup_{\mathbf{a} \in C} \sum_{n=N}^{\infty} |a_n| = 0$$

Q2. Characterize the compact subsets of the space ℓ_∞^0 of sequences $\mathbf{a} = \{a_n : n \geq 1\}$ with the property, $\lim_{n \to \infty} a_n = 0$, the metric being

$$d(\mathbf{a}, \mathbf{b}) = \sup_{n \geq 1} |a_n - b_n|$$