

Final Take Home Examination.

Due before the end of term.

Q1. Let $f(t) \geq 0$ be a weight on $[0, 2\pi]$. Let

$$\sigma^2 = \inf \left[\int_0^{2\pi} \left| 1 - \sum_{j \neq 0} a_j e^{ijt} \right|^2 f(t) dt \right]$$

be the error in approximating 1 by functions involving only non zero frequencies in $L_2(f dt)$. When is $\sigma^2 > 0$ and what is it?

Q2. The Riesz transforms map $L_\infty \rightarrow BMO$. Why do they map $BMO \rightarrow BMO$?

Q3. A function $f(x)$ on R is said to be in VMO if

$$\limsup_{h \rightarrow 0} \sup_x \left[\frac{1}{2h} \int_{x-h}^{x+h} |f(y) - \frac{1}{2h} \int_{x-h}^{x+h} f(z) dz| dy \right] = 0$$

Can you find a function in BMO that is not in VMO ? Show that uniformly continuous functions are in VMO and in fact are dense in VMO .

Q4. For the permutation group of n objects, besides $\pi(g) = 1$ and $\pi(g) = \sigma(g)$ the sign of the permutation g , show directly that there are no other 1 dimensional representations.

Q5. Can you find a function f such that both f and its Fourier transform \hat{f} have compact support? Given an arbitrary positive function $\phi(t)$ on $[0, \infty]$ decreasing to zero as $t \rightarrow \infty$ can you find f such that both f and \hat{f} satisfy

$$|f(x)| \leq C\phi(|x|)$$

$$|\hat{f}(x)| \leq C\phi(|x|)$$

for some finite constant C ?