Probability, Limit Theorems

Problem set 3. Due Oct 10, 2002

Q1. If $f(x)$ is a bounded lower semicontinuous function on R and $\mu_n \Rightarrow \mu$ show that

$$\int f(x)d\mu(x) \leq \liminf_{n \to \infty} \int f(x)d\mu_n(x)$$

Hint: Write $f(x) = \lim \uparrow f_n(x)$ an increasing limit of bounded continuous functions.

Q2. If $f(x)$ is bounded and continuous at every point of A^c and $\mu(A) = 0$ then show that whenever $\mu_n \Rightarrow \mu$

$$\lim_{n \to \infty} \int f(x)d\mu_n(x) = \int f(x)d\mu(x)$$