Probability, Limit Theorems

Problem set 1. Sept 19, 2002

For each probability measure μ on the Borel subsets of R^2 define the distribution function $F(a,b)$ by

$$F(a,b) = \mu\big[(x,y) : -\infty < x \leq a, -\infty < y \leq b\big]$$

Show that $F(\cdot,\cdot)$ satisfies the following conditions:

1. $F(a,b)$ is non-decreasing and right continuous in each variable a and b.

2.
$$\lim_{a\to-\infty} F(a,b) = 0 \text{ for each } b$$
$$\lim_{b\to-\infty} F(a,b) = 0 \text{ for each } a$$
$$\lim_{\substack{a\to\infty \\ b\to\infty}} F(a,b) = 1$$

3. For $-\infty < a < a' < \infty$ and $-\infty < b < b' < \infty$

$$F(a',b') - F(a,b') - F(a',b) + F(a,b) \geq 0$$

Conversely, show that any F satisfying these conditions arises from a unique probability measure μ.

Due by Sept 26.