Find the conjugate harmonic functions of

1) $u = x^2 - y^2 + 5x - 6y - \frac{y}{x^2+y^2}$

2) $u = e^x(x\cos y - y\sin x) + 2\sin x \sinh y + x^3 - 3xy^2 + y$

3) $u = \log(x^2 + y^2)$

Find the all conformal maps of $\{y > 0\}$ on to

1. $\{x < 0\}$
2. $\{y > 0; x^2 + y^2 < 1\}$
3. $\{x > 0, y > 0\}$
4. $0 < x < 1$
5. The equilateral triangle with base the interval $-1 < x < 1$ on $y = 0$ and vertex at $i\sqrt{3}$

Evaluate the integral

$$\int_\gamma \frac{dz}{z}$$

where γ is an arbitrary curve in C from 1 to 2 that does not pass through 0.

Evaluate the integrals

1. $\int_0^\infty x^{s-1}\cos x\, dx$
2. $\int_0^\infty x^{s-1}\sin x\, dx$
3. $\int_0^\infty \frac{\sin x}{x} dx$
4. $\int_{-\infty}^\infty \frac{1}{1+x^6} dx$

Construct meromorphic functions having singularities only as listed below.

1. Simple poles at 0 and ∞.
2. A pole of order 2 at ∞ and a simple pole at 0.
3. Essential singularities at 0, 1 and ∞.

An entire function $f(z)$ has simple zeros at the points 2^n for $n \geq 1$ and no others. $f(0) = 1$, $f'(0) = 0$ and $|f(z)| \leq Ae^{B|z|}$. Determine f.

An entire function $f = u + iv$ satisfies $|u(z)| \leq c_1 + c_2|z|^k$. Is f necessarily a polynomial of degree at most k?

Consider $f(z) = \sqrt{z(z-5)}$ in a neighborhood of $z = 9$ on the real axis with $f(9) = 6$. Can it be continued analytically along the circle $|z| = 9$? If it can and you go around once and return to 9 is the value $f(9)$ now 6 or -6?