Problems for the week of Oct 20. Due Oct 27

1. If $f(z) = u(x, y) + iv(x, y)$ is analytic in the entire plane show that it is an odd function, i.e. $f(z) + f(-z) = 0$ if and only if $v = 0$ on the real axis and $u = 0$ on the imaginary axis.

2. If $f(z)$ is analytic in $|z| \leq 1$ and $f(z)| = 1$ on $|z| = 1$ show that f is a rational function, i.e. of the form $\frac{P}{Q}$ where P and Q are polynomials.

3. The Fibonacci numbers c_n are defined by c_0, $c_1 = 1$ and $c_n = c_{n-1}+c_{n-2}$ for $n \geq 2$. Determine the function

$$c(z) = \sum_0^\infty c_n z^n$$

as a rational function and use it to find a formula for c_n.

4. Show that the Laurent series for $f(z) = (e^z - 1)^{-1}$ is of the form

$$f(z) = \frac{1}{z} - \frac{1}{2} + \sum_{k=1}^\infty (-1)^{k-1} \frac{B_k}{(2k)!} z^{2k-1}$$